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(r*g*)* T_O, T_1 AND T_2 AXIOMS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce the concept of (r*g*)* $T_{O,}$ $T_{1,}$ and T_2 axioms and study some of their properties

KEYWORDS: (r*g*)* Closed Sets, (r*g*)* Closure, (r*g*)*Continuous Maps, (r*g*)* Irresolute Maps**,** (r*g*)* Open Sets

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1. INTRODUCTION

N Levin [6] introduced the concept of generalized closed set in topological spaces. Since then many Topologist started defining different types of closed sets and utilized these concepts to the various notion of subsets and axioms. The Authors [11] have already introduced (r*g*)* closed set and investigated some of the properties. In this paper, using the concept of (r*g*)* closed set we introduce separation axioms like (r*g*)* T_o,(r*g*)* T_1 and (r*g*)* T_2 axioms and some of their properties are investigated.

2. PRELIMINARIES

Definition **2.1:** A subset A of a space X is called a (r*g*)*closed set [11] if cl(A) $\subseteq$ U whenever A$\subseteq$ U and U is r*g*- open. The complement of (r*g*)*closed set is (r*g*)*open.

Definition 2.2: A map f : (X, τ) $\rightarrow$ (Y, σ) is called (r*g*)*-continuous [12] if the inverse image of every closed set in (Y, σ) is (r*g*)*-closed in (X, τ).

Definition 2.3: A map f : (X, τ) $\rightarrow$ (Y, σ) is said to be a (r*g*)*-irresolute map [12] if $f^{-1}(V)$ is a (r*g*)*-closed set in (X, τ) for every (r*g*)*-closed set V of (Y, σ).

Definition 2.4: Let X be Topological space. Let A be a subset of X. (r*g*)*closure[13] of A is defined as the intersection of all (r*g*)* closed sets containing A.

Definition 2.5: A property P holding good for a topological space(X, $\Im$) and which is also holds good for every subspace of the topological space is called Hereditary property.

Definition 2.6 : Let X be a non empty set and $\Im$ be a family of subsets of X consisting of empty set φ and all those nonempty subsets of X whose compliments are finite, then $\Im$ is a topology for X and is called co-finite topology or finite compliment topology.

3. (r*g*)*T_O SPACE

Definition 3.1: A Topological space (X, ℑ) is said to be (r*g*)*T_o space if and only if for every distinct points x, y in X there exists a (r*g*)*open set G such that x∈ G, y ∉ G or y ∈ G but x ∉ G, or for distinct points x and y of X there exists a (r*g*)*open set G containing one of the points but not the other. This property is known as (r*g*)*T_o axiom.

Example 3.2: Discrete space is a (r*g*)*T_o space.

Since There exists a (r*g*)* open set {x} Containing x but not containing y which is distinct from x, and hence it is a (r*g*)*T_o space.

Example 3.3: Consider the indiscrete Topological space. The (r*g*)*open sets are φ, X. The (r*g*)*open set X contains x and also y. Thus there is no (r*g*)*open set which contain x but not containing y. Hence it is not a (r*g*)*T_o space.

Theorem 3.4: A Topological space (X, ℑ) is a (r*g*)*T_o space iff the (r*g*)*closures of distinct points are distinct.

Let (X, ℑ) be (r*g*)* T_o and let x, y ∈ X with x ≠ y.

TPT (r*g*)*cl {x} ≠ (r*g*)*cl {y}

Since (X, ℑ) is (r*g*)*T_o there exists (r*g*)*open set G such that x ∈ G y ∉ G. Since G is (r*g*)*open, X –G is (r*g*)*closed. Also x ∉ X-G and y ∈ X – G

Now (r*g*)*cl{y} [13] is the intersection of all (r*g*)*closed sets containing y. Hence

y∈ (r*g*)*cl {y} but x ∉ (r*g*)* {y}. ∴ (r*g*)*cl {x} ≠ (r*g*)*cl {y}.

Conversely Let x ≠ y and (r*g*)*cl{x} ≠ (r*g*)*cl {y}

TPT (X, ℑ) is a (r*g*)* T_o space. Since (r*g*)* {x} ≠ (r*g*)* {y} there exists at least one point

z ∈ X such that z ∈ (r*g*)*cl { x} but z ∉ (r*g*)*cl{y}

Claim x ∉(r*g*)*cl {y}

If x ∈ (r*g*)*cl {y} then {x} ⊂(r*g*)*cl {y}

⇒ (r*g*)*cl{x} ⊂ (r*g*)*cl (r*g*)*cl{y}

⇒ (r*g*)*cl{x} ⊂(r*g*)*cl{y}

Therefore z ∈ (r*g*)*cl {x} ⇒ z ∈ (r*g*)*cl {y} Which is a contradiction ⇒ x ∉ (r*g*)*cl {y}

Now x ∉(r*g*)*cl{y} ⇒ x ∈$\{$(r*g*)*cl {y}$\}^c$ which is (r*g*)*open, Since (r*g*)*cl{y} is (r*g*)*closed. Thus $\{$(r*g*)*cl {y} $\}^c$ is (r*g*)*open set containing x but not y. Hence (X, ℑ) is a (r*g*)*T_o space.

Theorem 3.5: (Heriditary property) Every subspace of a (r*g*)*T_o space is a (r*g*)*T_o.

Proof: Let (X, $\Im$) be a $(r^*g^*)^*T_o$ space and (Y, $\Im^*$) be a subspace of (X, $\Im$) where $\Im^*$ is relative topology and $\Im^*=\{ Y \cap G : G \in \Im \}$

Let y_1, y_2 be two distinct points of Y. since $Y \subset X$, $y_1, y_2 \in X$. Since X is $(r^*g^*)^*T_o$ there exists a $(r^*g^*)^*$open set G such that $y_1 \in G$ and $y_2 \notin G$ By definition of subspace, $G \cap Y$ is a $\Im^*$ open set which contains y_1 but does not contain y_2. Hence (Y, $\Im^*$) is also a $(r^*g^*)^*T_o$ space.

4. $(r^*g^*)^*T_1$ SPACES

Definition 4.1: A Topological space (X, $\Im$) is said to be $(r^*g^*)^*$ T_1 space iff for any given pair of distinct points x and y there exists two $(r^*g^*)^*$ open sets G and H such that G contains x but not y and H contains y but not x. The above property is known as $(r^*g^*)^*T_1$ axiom.

Remark 4.2: Every $(r^*g^*)^*$ T_1 space is a $(r^*g^*)^*T_o$ space

Proof: From the definition of $(r^*g^*)^*$ T_1 space it follows that it is $(r^*g^*)^*T_o$, since there exists a $(r^*g^*)^*$open set G such that $x \in G$ but $y \notin G$

The Converse is not true.

Result 4.3: Every $(r^*g^*)^*T_o$ space is not a $(r^*g^*)^*$ T_1 space.

The following example Supports this.

Example 4.4: Consider the set N of all Natural numbers. Let $\Im$ be the collection consisting of φ, N and all those subsets of N of the form {1,2, ... n}, $n \in N$

The space (N, $\Im$) is a $(r^*g^*)^*T_o$ space

For, consider 2 distinct points m and n such that $m < n$.

G= {1,2,. . . .m} is an open set and hence $(r^*g^*)^*$open set containing m but not containing n and hence it is a $(r^*g^*)^*T_o$ space

But it is not T_1, because if we choose a $(r^*g^*)^*$open set H = {1,2,. . . n} then $m \in G$, $n \notin G$. But $n \in H$ and also $m \in H$ ($m < n$)

Hence (N, $\Im$) is not a $(r^*g^*)^*$ T_1 space. But it is a T_o space.

Theorem 4.5: (Hereditary property) Every subspace of a $(r^*g^*)^*$ T_1 space is $(r^*g^*)^*$ T_1 space.

Proof: Let (X, $\Im$) be a $(r^*g^*)^*T_1$ space and let (Y, $\Im$ *) be a subspace where $\Im$ * is a relative topology and $\Im^*=\{ Y \cap G : G \in \Im \}$

Let y_1, y_2 be two distinct points of Y. since $Y \subset X$ they are also points of X.

If G and H be two $(r^*g^*)^*$open subsets of X then $G^* = G \cap Y$ and $H^* = H \cap Y$ are $(r^*g^*)^*$ open subsets of Y and hence $y_1 \in G \Rightarrow y_1 \in G^*$ $y_1 \notin G \Rightarrow y_1 \notin G^*$

Since (X, $\Im$) is $(r^*g^*)^*$ T_1 space and y_1, y_2 are distinct points of X and Y we have

$y_1 \in G$ but $y_2 \notin G$ $\therefore$ $y_1 \in G^*$ but $y_2 \notin G^*$.

$y_2 \in H$ but $y_1 \notin H \therefore y_2 \in H^*$ but $y_1 \notin H^*$

Hence (Y, $\Im$ *) is also a (r*g*)*T_1 space.

Theorem 4.6: A Topological space (X, $\Im$) is a (r*g*)*T_1 space iff every singleton subset {x} of X is a (r*g*)*closed set.

Proof: Let Every singleton set {x} of X be (r*g*)*closed

TPT (X, $\Im$) is a (r*g*)*T_1 space

Since {x} is (r*g*)*closed, $\{x\}^c$ is (r*g*)*open

Let x,y be two distinct points of X so that {x} and {y} are (r*g*)* closed and $\{x\}^c$ and $\{y\}^c$ are (r*g*)* open. Now $y \in \{x\}^c$ but $x \notin \{x\}^c$. Now $x \in \{y\}^c$ but $y \notin \{y\}^c$

Hence (X, $\Im$) is a (r*g*)* T_1 space.

Conversely Let (X, $\Im$) be a (r*g*)* T_1 space. Let x be any arbitrary point of X.

TST {x} is (r*g*)*closed or $\{x\}^c$ is (r*g*)*open

Let $y \in \{x\}^c$ then $y \neq x$. Since the space is (r*g*)* T_1 and $y \neq x$, there must exist a (r*g*)*open set G_y such that $y \in G_y$ but $x \notin Gy$

Thus corresponding to each $y \in \{x\}^c$ there exists a (r*g*)*open set G_y such that

$y \in G_y \subset \{x\}^c \therefore \cup \{y / y \neq x\} \subset \cup \{Gy : y \neq x\} \subset \{x\}^c$

That is $\{x\}^c \subset \cup \{Gy : y \neq x\} \subset \{x\}^c$

$\Rightarrow \{x\}^c = \cup \{G_y : y \neq x\}$

Since G_y is (r*g*)*open $\cup \{G_y : x \neq y\}$ is (r*g*)* open. $\therefore \{x\}^c$ is (r*g*)*open and hence {x} is (r*g*)*closed

Since x is arbitrary it follows that every singleton subset {x} of X is (r*g*)*closed.

Cor 4.7: A Topological space (X, $\Im$) is (r*g*)* T_1 iff every finite subset of X is (r*g*)*closed.

Proof 4.8: Since a finite subset of X is the Union of finite number of singleton sets, is closed and hence (r*g*)*closed. Hence if (X, $\Im$) is (r*g*)* T_1 then every finite subset of X is (r*g*)*closed.

Conversely if every finite subset is (r*g*)*closed then as a particular case every singleton subset {x} being finite is also (r*g*)*closed.

Hence the space (X, $\Im$) is a (r*g*)* T_1 space.

Hence we can summarize the above results as follows.

Theorem 4.9: Let (X, $\Im$) be a Topological space, then the following statements are equivalent.

(a) (X, $\Im$) is a (r*g*)* T_1 space

(b)Every singleton subset of X is (r*g*)*closed.

(c)Every finite subset of X is (r*g*)*closed.

5. (r*g*)* T_2 SPACE OR (r*g*)* HAUSDORFF SPACE.

A Topological space (X, ℑ) is said to be a (r*g*)*T_2 space or (r*g*)* Hausdorff space if for every pair of distinct points x_1,x_2 of X there exist disjoint (r*g*)* open sets U and V of x_1 and x_2 respectively and such that U ∩ V = φ

Example 5.1: Discrete topological space. Let x and y be two distinct points so that

{ x} and {y} are open sets and hence (r*g*)*open sets of x and y such that

{x} ∩ {y}= φ. Hence X is a (r*g*)*T_2 space or (r*g*)* Hausdorff space.

Example 5.2: Indiscrete space.

Since there is only one non empty (r*g*)*open set, no two distinct points can have disjoint (r*g*)*open sets. Therefore X is not a (r*g*)*T_2 space or (r*g*)* Hausdorff space.

Theorem 5.3: (Hereditary property). Every subspace of (r*g*)* T_2 space is a (r*g*)*T_2 space.

Proof: Let (X, ℑ) be a (r*g*)*T_2 space and (Y, ℑ*) be a subspace of (X, ℑ) where ℑ* is the relative topology and ℑ*={ Y ∩ G : G ∈ ℑ }

Let x, y ∈ Y be distinct. They are also distinct points of X. Since X is (r*g*)*T_2 there exist

disjoint (r*g*)* open sets G and H of x & y respectively such that G ∩ H = φ

[Now x ∈ G, x ∈ Y ⇒ x ∈ Y ∩ G

y ∈ H, y ∈ Y ⇒ y ∈ Y ∩ H]

Now (Y ∩ G) ∩ (Y ∩ H) = Y ∩ (G ∩ H) = Y ∩ φ = φ

Hence Y ∩ G and Y ∩ H are (r*g*)*open sets of x and y relative to Y respectively

so that (Y, ℑ*) is also (r*g*)*T_2.

Example 5.4: X={a,b,c} ℑ ={ φ, X,{a},{ b,c}}

(r*g*)* Open sets { φ, X,{b,c},{a,c},{a,b},{c},{a}}

For a, b {a} ∩ {b,c} = φ

For b, c {a, b} ∩ {c} = φ

For a, c {a} ∩ {c} = φ. The space is (r*g*)*T_1 and hence (r*g*)* T_2 also.

X is a (r*g*)* T_o, T_1 T_2 but (X, ℑ) is not T_o, T_1, and T_2 but if the space is T_o, T_1, and T_2 then it is (r*g*)* T_o,T_1, and T_2 respectively.

Theorem 5.5: Every subset consisting of exactly one point of (r*g*)*T_2 space (X, ℑ) is (r*g*)*closed (or) Every singleton subset {x} of a (r*g*)*T_2 space is(r*g*)* closed.

Corollary 5.6: A finite subset of (r*g*)*T_2 space being the finite Union of singleton sets (which are (r*g*)*closed) is also (r*g*)*closed set.

Result 5.7: Every (r*g*)*T_2 space is a (r*g*)*T_1 space.

Proof: Let (X, ℑ) be a (r*g*)*T_2 space. Therefore there exists (r*g*)*open sets G and H containing x and y respectively such that G ⋂ H = φ

x ∈G, y ∈ H Also since x ∈ G, x ∉ H since G ⋂ H = φ y ∈ H ⇒ y ∉ G

Therefore x ∈ G, but y ∉ G and y ∈ H but x ∉ H. Hence the space (X, ℑ) is a (r*g*)*T_1 space.

But the converse is not true.That is Every (r*g*)* T_1 space is not necessarily a (r*g*)* T_2 space.

The converse need not be true as seen from the example.

Example 5.8: Let ℑ be the co-finite topology for an infinite set X. Then (X,ℑ) is a (r*g*)* $T_{1.}$ But not (r*g*)*T_2

Theorem 5.9: Let (X, $ℑ_1$) be Topological space and (Y, $ℑ_2$) be a (r*g*)*T_2 space and f : X → Y be a one – one,(r*g*)* irresolute map then (X, ℑ) is also a (r*g*)*T_2 space.

Proof 5.10: Let x_1 and x_2 be two distinct points of X so that $f(x_1) = y_1$ and $f(x_2) = y_2$ are distinct points of Y since f is 1 – 1. But (Y, $ℑ_2$) is (r*g*)*T_2 space. Hence corresponding to distinct points y_1 and y_2 of Y there exists (r*g*)*open sets G and H such that G ⋂ H = φ.

$y_1 = f(x_1) \in G$, $y_2 = f(x_2) \in H$, G ⋂ H = φ

$x_1 \in f^{-1}(G)$, $x_2 \in f^{-1}(H)$. Also $f^{-1}(G_1) \cap f^{-1}(H) = f^{-1}(G \cap H) = \varphi$

Since f is (r*g*)*irresolute, f^{-1}(G) and f^{-1} (“H) are(r*g*)*open in (X, $ℑ_1$) By definition of (r*g*)*T_2, (X, $ℑ_1$) is a (r*g*)*T_2 space.

Theorem 5.11: A Topological space (X, ℑ) is a (r*g*)*T_2 space iff each point x is the intersection of all (r*g*)* closed sets containing it.

Assume that (X, ℑ) is (r*g*)*T_2 There exists open sets G and H x ∈ G, y ∈ H and G ⋂ H =φ

Now G ⋂ H = φ G ⊂ X- H. Now x ∈ G ⊂ X – H so that X – H is (r*g*)* closed set containing x which does not contain y as y ∈ H. ∴ y cannot be contained in the intersection of all (r*g*)* closed sets of X which contain x. Since y ≠ x is arbitrary it follows that the intersection of all (r*g*)*closed sets containing x is {x}.

Conversely Let {x} be the intersection of all (r*g*)* closed subsets of X containing x where x is any arbitrary point of X, Let y be any other point of X which is different from x. Now y does not belong to the intersection. Therefore there exist a (r*g*)* closed set N such that y ∉ N.

Now {x} is (r*g*)* closed. X-{x} is (r*g*)*open, y ∈ X-N which (r*g*)* open

Now{x}= ⋂ N, N is a (r*g*)* closed set containing x implies N^c is an open set containing y.

Let y be any other point y ≠ x. Now {y}= ⋂ M where M is a (r*g*)* closed set. M^c is a(r*g*)* open set containing x. That is M^c and N^c are the two open sets containing x and y respectively. Hence the theorem.

Theorem 5.12: A Topological space (X, ℑ) is (r*g*)*T_2 iff for any two distinct points x and y there is a (r*g*)*open set N_y of y such that $x \notin$ (r*g*)* cl N_y

Proof 5.13: Let x and y be two distinct point of X. since X is (r*g*)*T_2 there exists (r*g*)*open sets

G and H such that $x \in G$, $y \in H$ and $G \cap H = \varphi$. Now $G \cap H = \varphi \Rightarrow H \subset X - G$ where X – G (r*g*)* is a closed set ∴ $y \in H \subset X - G$. Let X –(r*g*)*cl G = N_y then

(r*g*)*cl Ny = (r*g*)*cl X –(r*g*)*cl(r*g*)*cl G = X— (r*g*)*clG =Ny implies $x \notin$(r*g*)*cl N_y

Conversely Let as assume that for each $x,y \in X$ there exists (r*g*)*open set N_y of y and $x \notin$ (r*g*)*cl N_y .We know that $N_y \subset$(r*g*)*cl N_y Now N_y is (r*g*)*open and (r*g*)*clN_y is (r*g*)*closed. Now X- (r*g*)*cl N_y is (r*g*)*open. Also $x \notin$ (r*g*)*cl $N_y \Rightarrow x \in$ X -(r*g*)*clN_y

Thus we have two (r*g*)*open sets X- (r*g*)* cl N_y and N_y such that $x \in$ X - (r*g*)*cl N_y, $y \in N_y$ and such that (X-(r*g*)*cl N_y) $\cap N_y = \varphi$.

Hence (X, ℑ) is (r*g*)*T_2.

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